Standard model in a Nutshell

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Abstract

Understanding the complexity of the Standard Model (SM) of particle physics is crucial for young students aiming to pursue their future higher studies in physics. One way to comprehend the 'jigsaw puzzle' of the SM is to closely inspect its historical development throughout the twentieth century. This review offers a quick glance to some important pieces of the puzzle with proper references pointing them to detail theoretical formulations.

General formulation

SM is an unified quantum field theory of three of the four fundamental forces in nature, i.e., electromagnetic, strong and weak interactions. In Quantum Field Theory (QFT) both matter and interactions are represented by quantised fields (e.g. 'matter field' and 'interaction field'). An elementary particle is nothing else but simply the quantum excitation (or quantum) of the corresponding field. Any particle interaction in QFT is depicted as an interaction among corresponding 'fields' themselves.

The fundamental fields of the standard model are

- **Fermion Fields**, \( \psi \): Fermions, i.e., quarks, leptons and their antiparticles are the quanta of 'matter fields'. They are the spin \( \frac{1}{2} \) particles that build up the material contents of the universe. Fermions appear in three generations or families.

  \[
  \begin{pmatrix}
  e \\
  \nu_e \\
  \mu \\
  \nu_\mu \\
  \tau \\
  \nu_\tau
  \end{pmatrix}
  \]

  (1)

  and those for quarks are

  \[
  \begin{pmatrix}
  u \\
  d \\
  c \\
  s \\
  t \\
  b
  \end{pmatrix}
  \]

  (2)

- **Gauge Boson Fields**: These are the (vector) fields of three fundamental interactions- electromagnetic, weak and strong. The corresponding quanta are the spin 1 'Gauge boson' particles which are the 'force carriers' of the fundamental interactions (Table 1).

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Gauge Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic</td>
<td>( \gamma )</td>
</tr>
<tr>
<td>Weak</td>
<td>( W^\pm ) and ( Z^0 )</td>
</tr>
<tr>
<td>Strong</td>
<td>gluons</td>
</tr>
</tbody>
</table>

  • **Higgs Field**: This is a (scalar) quantum field \( \phi \), interaction with which gives a particle the mass. The quantum excitation of Higgs field is known as Higgs boson (spin 0).

  All of these standard model particles (i.e., fermions, gauge bosons and Higgs boson) have been experimentally verified to exist.

  In QFT, we can not directly measure the quantum fields or their 'internal configurations'. For example, excitation of an 'unobservable' electron field produces an electron, which has some 'observable' properties (charge, energy etc.) which we can actually measure. If any change (i.e. transformation) of the 'internal configurations' of a quantum field leaves the observable quantities invariant, we call the transformation a gauge transformation. Unlike global symmetry [1], the two states related by a gauge transformation are not actually different- the transformation just changed the internal configurations and reduced the degrees of freedom of the system. Such symmetries are called (local) gauge symmetries and follow Ward–Takahashi identity [2,3]. This identity is the quantum analogue of Noether's theorem [4], where the symmetries in a quantum field results into additional constraints or equations of motions (i.e. reduction of degrees of freedom).
The mathematical description of gauge symmetries is known as gauge theory, which was first introduced by Hermann Weyl in 1918 and further developed by Yang and Mills in '50s. Any theory involving symmetries can be further represented in terms of groups [5]. Gauge interactions in standard model thus can be described in terms of three symmetry groups:

\[ SU(3) \times SU(2) \times U(1) \]  

(3)

where the fields represented by \( SU(3) \), \( SU(2) \) and \( U(1) \) are summarised in Table 2.

Quark mixing and weak CP violation

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### Table 2: Gauge groups of standard model.

<table>
<thead>
<tr>
<th>Group</th>
<th>Fields before EWSB†</th>
<th>Fields after EWSB†</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(3)</td>
<td>( w_{\mu,2,3} )</td>
<td>( w_{\mu,2,3}^{\pm} )</td>
</tr>
<tr>
<td>SU(2)</td>
<td>( B_{s} )</td>
<td>( A_{s} )</td>
</tr>
<tr>
<td>U(1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Quark mixing and weak CP violation

An intriguing success of SM was the early predictions of many particles with correct properties long before they were experimentally observed. Among them worth mentioning the revolutionary quark model, developed by Gell–Mann and \cite{6,7} in 1964. Originally, they proposed three quark flavours—up, down and strange. The first indirect experimental hint of their existence came through Stanford Linear Accelerator Center (SLAC)'s deep inelastic scattering experiments \cite{4,5} near the end of the same decade. A fourth quark named 'charm' was proposed in 1970, by Glashow, Iliopoulos and Maiani, who extended Cabibo's 1963 idea of 'quark mixing' \cite{8} to reconcile the discrepancies in experimental and calculated rates of \( K \to \pi^0 \) decay. Their combined method of quark mixing is known as Cabibbo–GIM mechanism. Four years after the prediction, in 1974, C.C.Ting and his team discovered a new 'long-lived' particle, the \( W \) meson, which is, indeed, identified as a bound state of charm quark and its antiparticle (\( \nu = c\bar{c} \)).

Same year as the charm quark was discovered, Kobayashi and Maskawa predicted a third generation of quark by generalising the Cabibbo–GIM mechanism. After four years, the bottom quark was finally discovered in 1977 by E288 experiment at Fermilab \cite{10}. It took, however, 12 more years to discover its partner, the top quark in 1995 at Fermilab \cite{1,11}. Kobayashi and Maskawa jointly won a nobel prize for their contribution later in 2008.

**CKM matrix**

Weak interaction works in a different way for quarks than for the leptons. For instance, there is no cross-generation charged weak couplings (e.g. \( e^- \to \nu_{\mu} + W^- \)), but no such restriction exists for quarks (e.g. \( s \to u + W^- \) is possible). Secondly, the weak coupling constant for quarks carries an extra factor (sine or cosine of some angle). This can be explained in terms of the so called quark mixing. The key idea here is that the eigenstates (\( d', s', b' \)) involved in weak interaction are not the same as the physical 'mass' (\( d, s, b \)) eigenstates, rather the former ones are a linear superposition of the latter. The \( W^- \) s couple with the 'weak interaction' states

\[
\begin{pmatrix}
  u \\
  d' \\
  c \\
  s' \\
  t \\
  b'
\end{pmatrix}
\]

(4)

in exactly same way they do with the physical leptons. The weak and mass eigenstates of quarks are related by a 3×3 CKM matrix (\( V_{CKM} \)):

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix} = V_{CKM}
\]

(5)

**Weak CP violation**

Gell–mann \cite{12} and Pais \cite{13} pointed out in the early '50s that neutral kaon can convert into its own antiparticle

\[
K^0 \leftrightarrow \bar{K}^0
\]

(6)

and hence the neutral kaons we observe in laboratory are not \( K^0 \) or \( \bar{K}^0 \), but a mixture of two. The quantum superposition of \( K^0 \) or \( \bar{K}^0 \) eigenstates are

\[
K_1 = \frac{1}{2}(K^0 - \bar{K}^0) \quad \text{and} \quad K_2 = \frac{1}{2}(K^0 + \bar{K}^0)
\]

(7)

and as Kaons have negative intrinsic parities, CP operation on these states results in

\[
CP(K_1) = K_1 \quad \text{and} \quad CP(K_2) = -K_2
\]

(8)

Before 1964 discovery of weak CP violation \cite{14} by Cronin and Fitch, it was believed that CP (charge conjugation and parity) symmetry is universal. And if CP is conserved, we can expect the \( K_1 \to 2\pi \) process but \( K_2 \) should only decay by \( K_2 \to 3\pi \). Moreover, faster 2\( \pi \) decay will make \( K_2 \) a short–lived state, while \( K_1 \) should be the longer lived one. But Surprisingly, Cronin and Fitch observed a 2\( \pi \) decay of the longer lived component of the neutral kaon:

\[
008
\]

Now the key idea in BEH mechanism is the inclusion of a single complex scalar field ($SU(2)$ doublet) to the theory, i.e., the BEH field (or simply the Higgs field) $\Phi$. Consequence of such an inclusion is a vacuum expectation value of $\Phi$ which in turn spontaneously breaks the electroweak symmetry into the electromagnetic subgroup $U(1)_{EM}$:

$$SU(2)_Y \times U(1)_Y \rightarrow U(1)_{EM}$$

and generates two massive charged vector bosons ($W^\pm$), one neutral massive vector boson ($Z^0$) and a massless photon $\gamma$. This missing piece of the standard model, i.e., the Higgs boson was finally observed experimentally in 2012 at LHC [22] and thus completed the standard model.

**SM Lagrangian**

The dynamics of any physical interaction in standard model is expressed in terms of the ‘Lagrangian density’ or simply ‘Lagrangian’, $(\mathcal{L})$ of the system. In compact notation, the standard model Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{WZ}}^\text{mass}$$

The **Gauge term** $\mathcal{L}_{\text{gauge}}$ encodes information on three fundamental gauge bosons- photons, gluons, $W^\pm$ and $Z^0$ bosons and their mutual interactions (i.e., gluon–gluon, weak–weak, and weak–photon interaction). It can be expressed as

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \varepsilon_{\mu\nu} B^\mu B^\nu - \frac{1}{8} \varepsilon_{\mu\nu\rho\sigma} (W^\mu W^\nu W^\rho W^\sigma) - \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} (G^\mu G^\nu G^\rho G^\sigma)$$

where the symbols $B$ and $W$ denotes the field strength tensors for weak hypercharge and weak isospin fields respectively. $G$ denotes the field strength tensor of gluon field. $\mu$, $\nu$, $\rho$, and $\sigma$ are Lorentz indices and Einstein’s summation convention is to be used. The traces are over 8 gluon colour states and three weak charged and neutral states ($W$ and $Z$) for gluon and weak fields respectively.

The **Interaction term** $\mathcal{L}_{\text{int}}$ describes the interaction between gauge bosons and fermions. In compact notation, it can be written as

$$\mathcal{L}_{\text{int}} = (i \bar{\psi} D \psi + h.c.)$$

where fermion (quarks and lepton) fields are denoted by $\Psi$ and the bar over $\Psi$ indicates the transpose conjugate. $D$ is the covariant derivative which represents the coupling of the corresponding gauge field to the fermions. There is a hermitian conjugate (h.c.) term which cancel out any complex terms arising from mathematical operations and thus ensure that the final Lagrangian is real. However the second term is self-adjoint and we can ignore/omit the h.c part.

Weak hypercharge $Y_\mu$ connects the electric charge $Q$ (electromagnetic sector) with the third component $T_3$ of weak isospin (weak sector):

$$Q - T_3 = \frac{1}{2} Y_\mu$$

and the corresponding field is the weak hypercharge field $B_\mu$.

$K_L \rightarrow \pi^+ + \pi^-$

If one assumes $K_S$ and $K_L$ to be the states $K_1$ and $K_2$ respectively, the observation (eq. 9) is clearly a violation of CP symmetry according to eq. 8, known as the direct CP violation. Another possibility is to consider $K_L$ or $K_S$ as a mixture of $K_1$ and $K_2$ states,

$$K_L = \frac{1}{\sqrt{1+\epsilon^2}} K_1 + \epsilon K_2 \quad \text{and} \quad K_S = \frac{1}{\sqrt{1+\epsilon^2}} K_1 - \epsilon K_2$$

with $\epsilon \approx 2.3 \times 10^{-3}$ (calculated from experimental result). Even if this is the case, CP symmetry appears to be violated. This type of CP violation is called indirect CP violation.

Both direct or indirect CP violation, however, can be explained via CKM mechanism of quark mixing [15]. A standard parametrization of $V_{CKM}$ matrix [16] is

$$V_{CKM} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}s_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13} \end{bmatrix}$$

where $c_{ij}$ and $s_{ij}$ are the cosine and sine of the Euler angles [5] $(\theta_j)$ respectively with $i$ and $j$ referring to the quark generation ($i < j = 1, 2, 3$). The complex phase factor $\delta$ is responsible for CP violation and known as the CP-violating phase. Measurement of this factor currently is an interesting field of research in experimental particle physics. The best known values can be found in [16].

**Electroweak unification and higgs mechanism**

Grand unification of all the fundamental interactions in nature has always been an active field of research in theoretical particle physics. The first major step towards it was the SM electroweak unification– an unified description of weak and Electromagnetic (EM) interactions. The unification energy is in the scale of $246 \text{ GeV}$, and was achievable at the extremely hot universe shortly after big bang (grand unification epoch). At lower energies (quark epoch), this unified electroweak force split into weak and electromagnetic interactions as we see today. This splitting, i.e., Electroweak Symmetry Breaking (EWSB) can be described in terms of Glashow, Salam and Weinberg (GSW) [17–19] model of electroweak unification and Brout–Englert–Higgs (BEH) [20,21]. Before EWSB, the unified electroweak theory could be represented as

$$SU(2) \times U(1)_Y$$

where $SU(2)$ and $U(1)_Y$ groups correspond to weak isospin (three gauge bosons $W_{\mu\nu}^{\pm}$) and weak hypercharge [6] (one gauge boson $B_\mu$) field respectively. Note that this $U(1)_Y$ field is different than the electromagnetic $U(1)_{EM}$ field which only appears after the symmetry breaking. Before EWSB, all these gauge bosons were massless.

$\theta_{13}$ is the original Cabibbo angle Cabibbo used in 1963.
The Yukawa term, $\mathcal{L}_Y$, describe the coupling of fermions with the BWH field $\phi$. This is the term that generates particle mass.

$$\mathcal{L}_Y = \bar{\psi}_i \gamma_j \psi_j \phi + h.c.$$ (18)

The hermitian conjugate (h.c.) term is necessary here, as it causes the mass of the antiparticles. Originally postulated by Yukawa to describe the nuclear force being mediated by pions, Yukawa interaction is now widely defined as an interaction between a scalar field (e.g. Higgs field) and a Dirac field (e.g. fermion field). This is why the third term on eq. 15 is known as Yukawa term and $Y_\nu$ are called Yukawa matrix.

The mass term of weak gauge bosons, $\mathcal{L}_{WZmass}$ describes the couplings of gauge bosons with BEH field. However, this term includes the gauge bosons $W^\pm$ and $Z^0$ of weak interactions only, as photons and gluons do not couple with the BEH field. Again, interaction with Higgs field generates the mass of $W^\pm$ and $Z^0$ bosons.

$$\mathcal{L}_{WZmass} = |D_\mu \phi|^2$$ (19)

The Higgs term, $\mathcal{L}_{\text{Higgs}}$, is fifth and final term which represents the vector BEH field $V$ and also includes the self-interaction of Higgs bosons, quanta of BEH field.

$$\mathcal{L}_{\text{Higgs}} = -V(\phi)$$ (20)

We omitted two more terms in eq.15, the gauge fixing term $\mathcal{L}_G$ and the ghost term $\mathcal{L}_{\text{ghost}}$. They are some unphysical states introduced to fix up mathematical inconsistencies in the gauge theory.

Concluding remarks

There is no doubt that the SM is a successful theory of the particle world, but it is still not perfect. A plethora of phenomena exists (e.g. the neutrino oscillations, existence of dark matter and dark energy) that cannot be explained by the SM. This implies the unavoidable need of new physics beyond the SM.

References